Product-Sum Identities

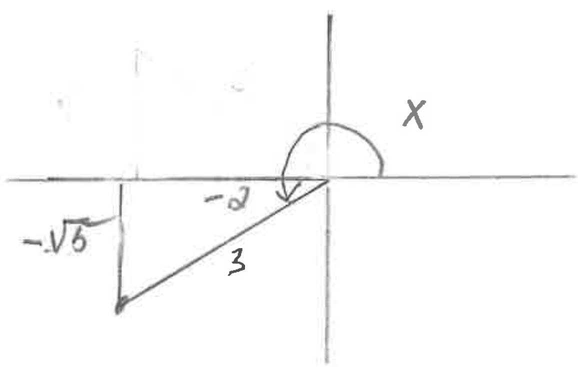
Double Angle Identities:

$sin(2\theta) = 2\,sin\,\theta\,cos\,\theta$

$cos(2\theta) = cos^2\theta - sin^2\theta \;=\; 2cos^2\theta - 1 \;=\; 1 - 2sin^2\theta$

$tan(2\theta) = \frac{2\,tan\,\theta}{1 - tan^2\theta}$

Given that $cos\,x = \; - \,\frac{2}{3}$ and that $sin\,x$ is negative, determine $cos\,2x,\; sin\,2x,$ and $\;tan\,2x.$



$sin\,x = \frac{-\sqrt{5}}{3}$

$tan\,x = \frac{\sqrt{5}}{2}$

$cos\,2x = cos^2x - sin^2x = \frac{4}{9} - \frac{5}{9} = -\frac{1}{9}$

$sin\,2x = 2\,sin\,x\,cos\,x = \frac{-2\sqrt{5}}{3}\cdot\left(\frac{-2}{3}\right) = \frac{4\sqrt{5}}{9}$

$tan\,2x = \frac{2\,tan\,x}{1 - tan^2x} = \frac{2\cdot\frac{\sqrt{5}}{2}}{1 - \frac{5}{4}} = -4\sqrt{5}$

Half Angle Identities:

$$sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cos\,x}{2}}$$

The sign depends on which quadrant $\frac{x}{2}$ is in.

$$cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+cos\,x}{2}}$$

$$tan\left(\frac{x}{2}\right) = \frac{1-cos\,x}{sin\,x}$$

$$tan\left(\frac{x}{2}\right) = \frac{sin\,x}{1+cos\,x}$$

Determine the exact value of $sin\,\frac{\pi}{12}$ without a calculator.

$\frac{\pi}{12} = \frac{\frac{\pi}{6}}{2}$, (first quadrant)

$$sin\frac{\pi}{12} = sin\frac{\frac{\pi}{6}}{2} = +\sqrt{\frac{1-cos\frac{\pi}{6}}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$$

Power Reducing Identities:

$sin^2 x = \frac{1}{2}(1 - cos\, 2x)$

$cos^2 x = \frac{1}{2}(1 + cos\, 2x)$

$tan^2 x = \frac{1 - cos\, 2x}{1 + cos\, 2x}$

Rewrite the expression $sin^5 x$ in terms of only first powers of sine and cosine.

$$\begin{aligned} sin^5 x &= (sin^2 x)^2 sin x \\ &= \left[\frac{1}{2}(1 - cos 2x)\right]^2 \cdot sin x \\ &= \frac{1}{4}(1 - 2cos 2x + cos^2 2x)\, sin x \\ &= \frac{1}{4}\left(1 - 2cos 2x + \frac{1 + cos 4x}{2}\right) sin x \end{aligned}$$

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Sum-to-Product Identities:

$sin\,x + sin\,y = 2\,sin(\frac{x+y}{2})\,cos\,(\frac{x-y}{2})$

$sin\,x - sin\,y = 2\,cos(\frac{x+y}{2})\,sin\,(\frac{x-y}{2})$

$cos\,x + cos\,y = 2\,cos(\frac{x+y}{2})\,cos\,(\frac{x-y}{2})$

$cos\,x - cos\,y = -2\,sin(\frac{x+y}{2})\,sin\,(\frac{x-y}{2})$

Write the following expression as a product

$sin\,105^o - sin\,50^o$

$= 2\cos\left(\frac{155^\circ}{2}\right)\sin\left(\frac{55^\circ}{2}\right)$

Product-to-Sum Identities:

$sin\, x\, cos\, y = \frac{1}{2}[sin\,(x+y) + sin\,(x-y)]$

$sin\, x\, sin\, y = \frac{1}{2}[cos(x-y) - cos(x+y)]$

$cos\, x\, cos\, y = \frac{1}{2}[cos(x+y) + cos(x-y)]$

Write the following expression as a sum instead of a product:

$4\,cos(\frac{3\pi}{5})\,sin(\frac{2\pi}{5}) \quad = 4\sin(\frac{2\pi}{5})\cos(\frac{3\pi}{5})$

$= 4 \cdot \frac{1}{2}\left[\sin\left(\frac{2\pi}{5} + \frac{3\pi}{5}\right) + \sin\left(\frac{2\pi}{5} - \frac{3\pi}{5}\right)\right]$

$= 2\sin(\pi) + 2\sin\left(\frac{-\pi}{5}\right)$

$= 0 - 2\sin\left(\frac{\pi}{5}\right) = -2\sin\left(\frac{\pi}{5}\right)$